[Project] The motion of a rotary motor is described by the overdamped Langeving equation

$$\frac{d\theta}{dt} = -\frac{dV(\theta)}{d\theta} + f_0 + \eta(t),$$

where $0 \leq \theta < 2\pi$ is the angular position of the motor, $V(\theta) = cos\theta$ is the periodic potential, and f_0 is the constant applied torque.
(1) Write down the corresponding Fokker-Planck equation for the probability distribution function $P(\theta, t)$ and solve it to find the stationary state distribution $P_{ss}(\theta)$. Compare it with the equilibrium Boltzmann distribution.
(2) Find the average angular velocity $\Omega = \lim_{t\to\infty} \frac{1}{t}\langle(\theta(t) - \theta(0))\rangle$ as a function of the applied torque f_0.
(3) Prepare your system in the equilibrium state with f_0 and then turn on the torque at time $t = 0$. Solve the Langevin equation numerically upto time $t = \tau$ and measure the work done by the external torque

$$W(\tau) = \int_0^\tau f_0\dot{\theta}(t')dt' .$$

By performing the numerical simulations many times, you can construct the probability distribution function for the work $P(W)$. Test the integral and detailed fluctuation theorems by computing $\langle e^{-W}\rangle$ and $P(W)/P(-W)$.